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Exact Statistical Inferences about the Parameter for an Exponential Growth Curve following a Poisson Distribution

J. K. LINDSEY[1]

Fisheries Research Board of Canada
Biological Station, Nanaimo, B.C.

LINDSEY, J. K. 1970. Exact statistical inferences about the parameter for an exponential growth curve following a Poisson distribution. J. Fish. Res. Bd. Canada 27: 172–174.

A use of the relative likelihood function is described. An example is provided to show exact statistical inferences about the parameter for an exponential increase in cell concentration when the concentration is a Poisson random variable.

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SUPPOSE that the concentration of some organism increases exponentially from some known starting level P_0 at time $t = 0$, to a level P_t at time t. Then,

[1]Present address: Department of Mathematics, Imperial College, University of London, London, England.

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$$P_t = P_0 e^{kt} + \text{error}.$$

If a small sample measuring concentration is removed from a larger culture, it may be assumed to have the Poisson distribution. Then the mean value of P_t is

$$E(P_t) = P_0 e^{kt} = \mu \tag{1}$$

and the distribution of P_t is

$$f(P_t; k) = \frac{e^{-\mu} \mu^{P_t}}{P_t!} = \frac{e^{-P_0 e^{kt}} P_0^{P_t} e^{ktP_t}}{P_t!}. \tag{2}$$

After a value of P_t has been observed, the likelihood function (see Sprott and Kalbfleisch, 1965; for more details see Barnard et al., 1962, and Sprott and Kalbfleisch, 1969) for k is proportional to

$$L(k) = \exp\{ktP_t - P_0 e^{kt}\}. \tag{3}$$

This function is maximized at the maximum likelihood estimate,

$$\hat{k} = \frac{\log P_t - \log P_0}{t}. \tag{4}$$

The precision of this estimate is given by the relative likelihood function

$$R(k) = \frac{\exp\{ktP_t - P_0 e^{kt}\}}{\exp\{\hat{k}tP_t - P_0 e^{\hat{k}t}\}} \tag{5}$$

which may be plotted for various values of k ($0 \leq R \leq 1$). Values of $R > 0.1$ might be considered to give a reasonable interval for k, comparable with a 95% probability interval for a normal theory linear model.

The use of the relative likelihood avoids the customary normal theory approximation; inferences using relative likelihoods are exact and in addition, in this example, require fewer calculations than the approximate methods. More complicated growth curves (e.g., von Bertalanffy curve) are amenable to the same analysis and are essentially as easy to deal with due to present day accessibility of high speed computers.

Suppose that the initial concentration of cells is measured as 20 cells per milliliter and that after 5 hr this has increased to 100 cells per milliliter. Then, $P_0 = 20$, $P_5 = 100$, $t = 5$

$$R(k) = \exp\{500k - 20e^{5k} - 60.944\}$$
$$\hat{k} = 0.32189$$

from equations 5 and 4. Values of the relative likelihood function are:

k	0.1	0.2	0.28	0.3	0.32189	0.36	0.37	0.38	0.4
R	10^{-19}	10^{-7}	0.117	0.475	1.0	0.143	0.0432	0.0237	10^{-3}

Then a 10% relative likelihood interval (Fig. 1) is about (0.278, 0.362).

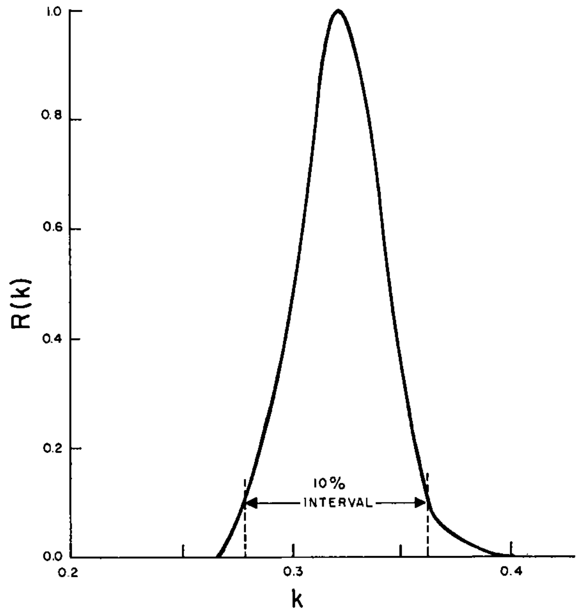

FIG. 1. Relative likelihood function for k.

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The Manefish, *Caristius groenlandicus* Jensen (Percomorphi: Caristiidae), in Atlantic Waters off Canada

W. B. SCOTT,[1] A. C. KOHLER,[2] AND R. E. ZURBRIGG[3]

SCOTT, W. B., A. C. KOHLER, AND R. E. ZURBRIGG. 1970. The manefish, *Caristius groenlandicus* Jensen (Percomorphi: Caristiidae), in Atlantic waters off Canada. J. Fish. Res. Bd. Canada 27: 174–179.

Morphometric and meristic data for the first recorded captures of *Caristius groenlandicus* in Canadian Atlantic waters in 1966 (1) and 1968 (7) are described and compared with previously published records.

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[1]Royal Ontario Museum and University of Toronto, Toronto, Ont.
[2]Fisheries Research Board of Canada, Biological Station, St. Andrews, N.B.
[3]Department of Zoology, University of Toronto, Toronto, Ont.

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